# ASSIGNMENT SET - I

**Department of Mathematics**

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**B.Sc Hon.(CBCS)**

**Mathematics: Semester-IV**

**Paper Code: C10T**

**[Linear Algebra-1]**

**Answer all the questions**

1. Determine wheather the following sets are subspace of $\mathbb{R}^3$ under the operations of addition and scalar multiplication on $\mathbb{R}^3$ . Justify your answer.

   (a) $W_1 = \{(a_1, a_2, a_3) \epsilon \mathbb{R}^3 : a_1 - 4a_2 + 7a_3 = 0\}$

   (b) $W_2 = \{(a_1, a_2, a_3) \epsilon \mathbb{R}^3 : 2a_1 - 8a_2 + 9a_3 = 0\}$

2. Determine whether the following sets are linearly dependent or linearly independent

   (a) $\{x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\}$ in $P_3(R)$

   (b) $\{x^3 - x, 2x^2 + 4, -2x^3 + 3x^2 + 2x + 6\}$ in $P_3(R)$

3. Determine which of the following sets are bases of $P_2(\mathbb{R})$

   (a) $\{1 + 2x - x^2, 4 - 2x + x^2, -1 + 18x - 9x^2\}$

   (b) $\{-1 + 2x + 4x^2, 3 - 4x - 10x^2, -2 - 5x - 6x^2\}$

4. Lebel the following statements as true or false.

(i) Any set containing zero vector is linearly independent.

(ii) Subsets of linearly independent sets is independent.

(iii)Every vector space has finite basis.

(iv)Every subspace of a finite dimensional space is finite dimensional.

5. Suppose that $T: \mathbb{R}^2 \to \mathbb{R}^2$ is linear, T(1,0)=(1,4) and T(1,1)=(2,5) then what is T(2,3)?Is T is one to one?

6. Let V and W be vector space of equal (finite) dimensional and let $T: V \to W$ be linear. Then the following are equivalent.

(a) T is one to one

(b) T is onto

(c) rank(T)=dim(V)

7. Let $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the linear transformation defined by $T\big(f(x)\big) = 2f'(x) + \int_0^x 3f(t)dt$

then find rank, nullity and matrix representation with respect to standard basis.

8. Let V be a vector space and S a subset that generates V. If β is a maximal linearly independent subset of S, then show that β is a basis for V.

9. Let V be a vector space that is generated by a set G containing exactly n vectors, and let L be a linearly independent subset of V containing exactly m vectors. Then m≤ n and there exist a subset H of G containing exactly n-m vectors such that $L \cup H$ generates V.

10. Show that every field is an integral domain but the converse is not necessarily true.

11. Prove that in a ring R if a is an idempotent element then 1-a is also an idempotent element.

12. Let I and J be two ideals of a ring R. Then $(I + J)$ and $I \cap J$ are also ideals and the factor ring $(I + J)$ and $I/_{(I+J)}$ are isomorphic.

13. Every ideal of the ring of integers $(\mathbb{Z}, +, .)$ is a principal ideal.

14. Show that the set of all R-valued functions defined on $[0,1]$ having the property $f(x) = f(1 - x)$ is a vector space over R.

15. Find the basis and dimension of the subspace W in $\mathbb{R}^3$ where

W= $\{(x, y, z)\epsilon\mathbb{R}^3: x + 2y + 3z = 0,5x - 3y + 4z = 0\}$

16. Let A and B be two subspaces of a finite dimensional vector space V. Then A+B is also finite dimensional, and

$\dim(A + B) = dimA + dimB - \dim(A \cap B)$

17.Let $T: U(F) \to V(F)$ be a linear transformation and U be a finite dimensional vector space then prove that

rank of (T)+nullity of (T)= dim (U)

18. Prove that a linear transformation $L: V \to W$ is non-singular if and only if the set $\{Lx_1, Lx_2, \dots, Lx_n\}$ is a basis of W whenever the set $\{x_1, x_2, \dots \dots, x_n\}$ is a basis of V.

19. The matrix m(T) of a linear mapping $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ relative to the ordered basis $\{(0,1,1), (1,0,1), (1,1,0)\}$ of $\mathbb{R}^3$ and $\{(1,0), (1,1)\}$ of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$ then find T.

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